When Safety is Not Safe Enough

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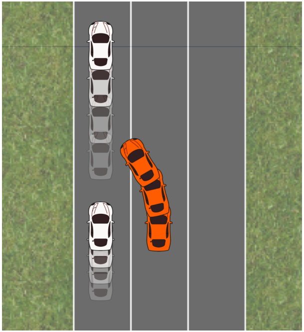
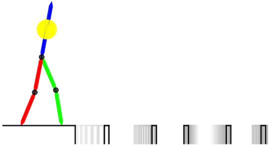
April 26, 2022

Motivation

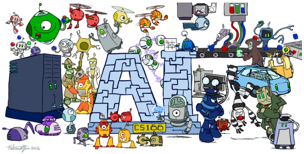


Credit: CS188



Credit: HRL

Credit: KiwiBot



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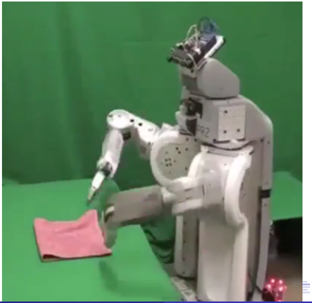

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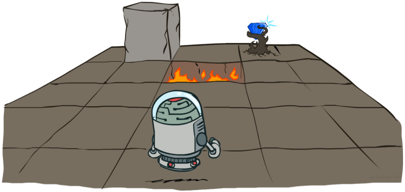

Courtesy: CS188

Safety is crucial in any engineering system

- driving safely on road without colliding with any object/ vehicle.
- Maintaining lane in autonomous vehicles.

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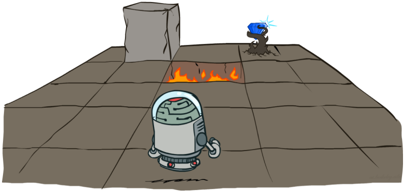

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Safety is crucial in any engineering system

- driving safely on road without colliding with any object/ vehicle.
- Maintaining lane in autonomous vehicles.
- Basically, any and every form of robot has some safety requirement

Safety $\equiv$ Set invariance

- Safety requirement can be cast as (in-)famous set-invariance
- Consider a dynamical system

$$\dot{x} = f(x) \quad x(0) = \bar{x}$$

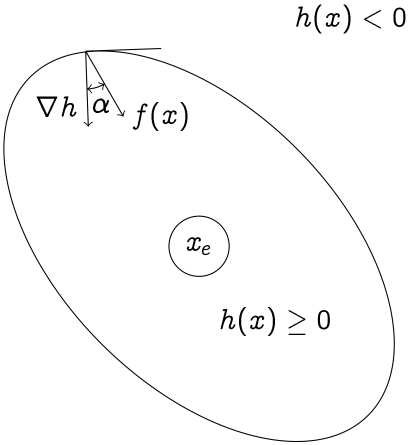
The safe set is defined by

$$\mathcal{C} = \{x : h(x) \geq 0\}$$
$$\partial\mathcal{C} = \{x : h(x) = 0\}$$

Then,

$$\mathcal{C} \text{ is invariant} \quad \iff \quad (\nabla h(x))^\top f(x) \geq 0 \quad \forall\ x \in \partial\mathcal{C}$$

Seeing is Believing



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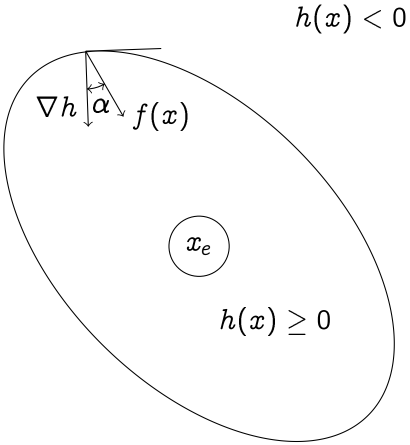

α is acute

Problem Setup

Consider a control affine nonlinear control system

$$\dot{x} = f(x) + \sum_{i=1}^{m} g_i(x) u_i \quad x(0) = \bar{x}$$

Let's characterize the safe set by

$$\mathcal{C} = \{x \in \mathbb{R}^n : h(x) \geq 0\}$$

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GOAL

1. Ensure that the trajectory is safe (i.e. $x(t) \in \mathcal{C}$ for $t \geq 0$)
2. The equilibrium, x_e , is asymptotically stable (i.e. $x(t) \xrightarrow{t \to +\infty} x_e$)

Guaranteed Stability

Consider the control system

$$\dot{x} = f(x) + \sum_{i=1}^{m} g_i(x)u_i \quad x(0) = \bar{x}$$

Lyapunov function

A continuously differentiable function $V : \mathbb{R}^n \to [0, +\infty[$ is called Control Lyapunov Function if

$$c_1 ||x||_2^2 \leq V(x) \leq c_2 ||x||_2^2$$

$$\inf_{u \in U} \left[\underbrace{L_f V + L_g V u}_{\dot{V}} + \gamma V \right] \leq 0$$

where $L_f V = (\nabla f)^\top V$, $L_g V = (\nabla g)^\top V$

Guaranteed Safety

For the control system

$$\dot{x} = f(x) + \sum_{i=1}^{m} g_i(x) u_i \quad x(0) = \bar{x}$$

and the safe set $\mathcal{C} = \{x : \mathbb{R}^n : h(x) \geq 0\}$

First safety certificate (Reciprocal Control Barrier Function)

A continuously differentiable function $B : \mathbb{R}^n \to \mathbb{R}$ is RCBF if

$$\frac{1}{\alpha_1(\|x\|_{\partial\mathcal{C}})} \leq B(x) \leq \frac{1}{\alpha_2(\|x\|_{\partial\mathcal{C}})}$$

$$\inf_u \Big[\underbrace{L_f B + L_g B u}_{\dot{B}} - \frac{\gamma}{B} \Big] \leq 0$$

Second safety certificate

For the control system

$$\dot{x} = f(x) + \sum_{i=1}^{m} g_i(x) u_i \quad x(0) = \bar{x}$$

and the safe set $\mathcal{C} = \{x : \mathbb{R}^n : h(x) \geq 0\}$

Zeroing Control Barrier Function

A continuously differentiable function $h : \mathbb{R}^n \to \mathbb{R}$ is ZCBF if

$$\sup_u \left[L_f h + L_g h u \right] \geq -\alpha(h(x))$$

Safe & Stable(almost)

The safety and stability objective are combined to form a QP

Safe and Stable controller

$$\begin{aligned} \min_{u} \quad & u^\top u \\ \text{subject to} \quad & L_f V + L_g V u + \gamma V \leq 0 \\ & L_f h + L_g h u + \lambda h \geq 0 \end{aligned}$$

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Safe and (almost)Stable controller

$$\begin{aligned} \min_{u,\delta} \quad & u^\top u + \delta^2 \\ \text{subject to} \quad & L_f V + L_g V u + \gamma V \leq \delta \\ & L_f h + L_g h u + \lambda h \geq 0 \end{aligned}$$

Safe & Stable(almost)

Safe and (almost)Stable controller

$$\min_{U=(u,\delta)} \quad U^\top U$$

$$\text{subject to} \quad \mathcal{A}U + \mathcal{B} \leq 0$$

where $\mathcal{A} = \begin{pmatrix} L_g V & -1 \\ -L_g h & 0 \end{pmatrix}, \mathcal{B} = \begin{pmatrix} L_f V + \gamma V \\ -L_f h - \lambda h \end{pmatrix}$

(Updated) Setup

Consider a control affine nonlinear control system

$$\dot{x} = f(x) + \sum_{i=1}^{m} g_i(x)u_i \quad x(0) = \bar{x}$$

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2. The equilibrium, x_e , is asymptotically stable (i.e. $x(t) \xrightarrow{t \to +\infty} x_e$)
3. System is robust to the perturbations in the control barrier function.

Guaranteed safety against finite “scenarios”

We assume that the $h \in \mathcal{H}_F = \{h_i\}_{i=1}^q$. We want the system to be robust against all such scenarios

Immune to any perturbation

$$\begin{aligned} \min_{u,\delta} \quad & u^\top u + \delta^2 \\ \text{subject to} \quad & L_f V + L_g V u + \gamma V \leq \delta \\ & \inf_{h \in \mathcal{H}_F} (L_f h + L_g h u + \lambda h) \geq 0 \equiv L_f h_i + L_g h_i u + \lambda h_i \geq 0 \ \forall \ i \end{aligned}$$

Specialized setting

For cleaner expressions we work with linear control systems

$$\dot{x} = \underbrace{Ax}_{f(x)} + \underbrace{B}_{g(x)} u$$

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The values of different certificates are taken to be

1. $V(x) = x^\top Px$, where $P \succ 0$
2. $h(x) = a^\top x + b$

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Safe and stable controller for Linear Control System

$$\begin{aligned} \min_{u,\delta} \quad & u^\top u + \delta^2 \\ \text{subject to} \quad & 2x^\top PAx + 2x^\top PBu + \gamma x^\top Px \leq \delta \\ & a^\top Ax + a^\top Bu + \lambda(a^\top x + b) \geq 0 \end{aligned}$$

Parametrized perturbations to safe set

Under the previous setting, let's assume that

$$a = \bar{a} + H\zeta \quad \text{where } ||\zeta||_2 \leq \rho$$

The resulting optimization problem which is immune to this perturbation

Safety against ellipsiodal perturbations in safe set

$$\begin{aligned} \min_{u,\delta} \quad & u^\top u + \delta^2 \\ \text{subject to} \quad & 2x^\top PAx + 2x^\top PBu + \gamma x^\top Px \leq \delta \\ & \bar{a}^\top Ax + \bar{a}^\top Bu + \lambda(\bar{a}^\top x + b) - \rho||H^\top Ax + H^\top Bu + \lambda H^\top x||_2 \geq 0 \end{aligned}$$

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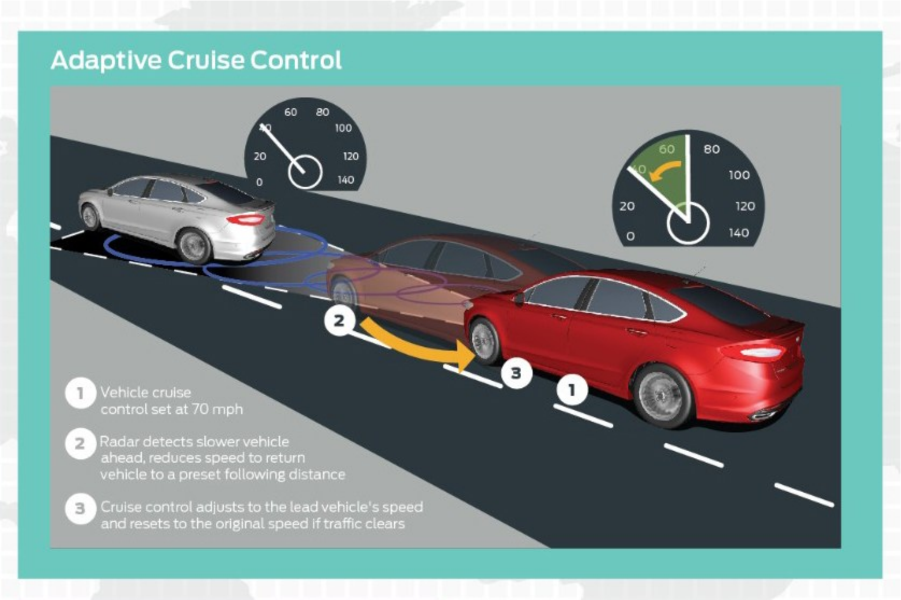
Thus, the problem reduced to solving for **SOCP**

Adaptive Cruise Control



Adaptive Cruise Control

Goal

1. Reach desired speed
2. Adjust vehicle speed to keep a safe distance from nearby vehicles.

Dynamics

$$m\frac{dv}{dt} = F_w - F_r$$

where

- F_w: Wheel force (control input u)
- F_r: Aerodynamic drag

Adaptive Cruise Control

Dynamics

In linear control system form:

$$\dot{x} = \begin{bmatrix} x_f \\ v \\ D \end{bmatrix} = \begin{bmatrix} v \\ -\frac{1}{m} F_r \\ v_0 - v \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{1}{m} \\ 0 \end{bmatrix} u$$

where

- x_f - Position of the follower
- v - Velocity of the follower
- v_0 - Velocity of the leader
- D - Distance between the follower and leader
- u - Force applied by the follower (F_w)

Adaptive Cruise Control

Stability Constraint

Use Lyapunov function $V(y) = y^2$ with where $y = v - v_d$. Applying this to

$$L_f V(x) + L_g V(x)u + \gamma V(x) - \delta \leq 0$$

yields

$$\psi_0(v) = -\frac{2(v - v_d)}{m} F_r + \epsilon(v - v_d)^2$$
$$\psi_1(v) = \frac{2(v - v_d)}{m}$$
$$\psi_0(v) + \psi_1(v)u \leq \delta$$

Adaptive Cruise Control

Safety Constraint

Keep safe distance from leader: Heuristic Approach

$$h(x) = D - cv \geq 0$$

Where $c = 1.8$. Applying to

$$L_f h(x) + L_g h(x) u + \alpha(h(x)) \geq 0$$

yields

$$-1.8\frac{F_r}{m} + (v_0 - v) + \frac{1.8}{m}u + \gamma h(x) \geq 0$$

Perturbations

We consider perturbations in c i.e. $c \in \{1.2, 1.8, 2.4\}$

Adaptive Cruise Control

Quadratic Programming

Objective will be minimizing μ^2 of $u = F_w = F_r + m\mu$. Formulating QP in terms of $\mathbf{u} = \begin{bmatrix} u \\ \delta \end{bmatrix}$ with soft and hard constraints becomes:

$$\min_{\mathbf{u}} \mathbf{u}^T H_{acc} \mathbf{u} + f_{acc}^T \mathbf{u}$$

With safety and stability constraints.Where

$$H_{acc} = \begin{bmatrix} \frac{1}{m^2} & 0 \\ 0 & p_{sc} \end{bmatrix}, \; f_{acc} = \begin{bmatrix} \frac{F_r}{m^2} \\ 0 \end{bmatrix}$$

Simulation results

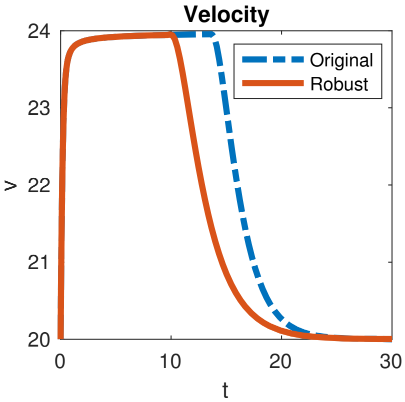


Figure: Velocity over time

- Reaches Velocity v_d quickly.
- Reduces speed when safety contraint becomes active.
- Robust case (perturbed constraints): reduce speed faster since it includes more conservative safety constraint

Simulation results

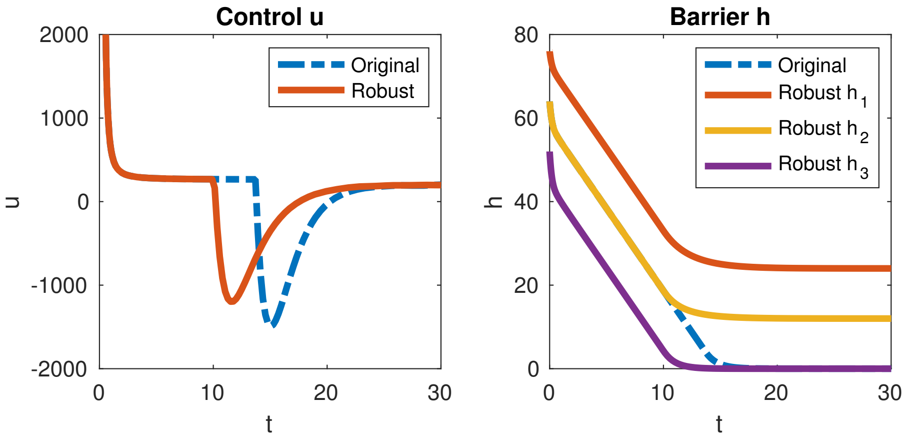


Figure: Control values and the safety certificate

Avoiding a circular set

Goal : Reach destination while avoiding obstacles.

Avoiding a circular set

- System Dynamics

$$\dot{x} = x + u, \quad x, u \in \mathbb{R}^2$$

- Safety Constraint

$$h(x) = -(||x - c||_2^2 - r_1^2)(||x - c||_2^2 - r_2^2)$$

- Stability Constraint

$$V(x) = x^T x$$

- Then we solve QP

$$\min_u u^T u$$

with safety and stability constraints converted to linear inequality form.

Avoiding a circular set

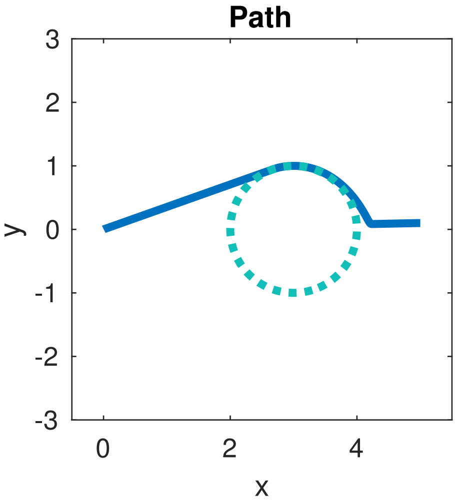
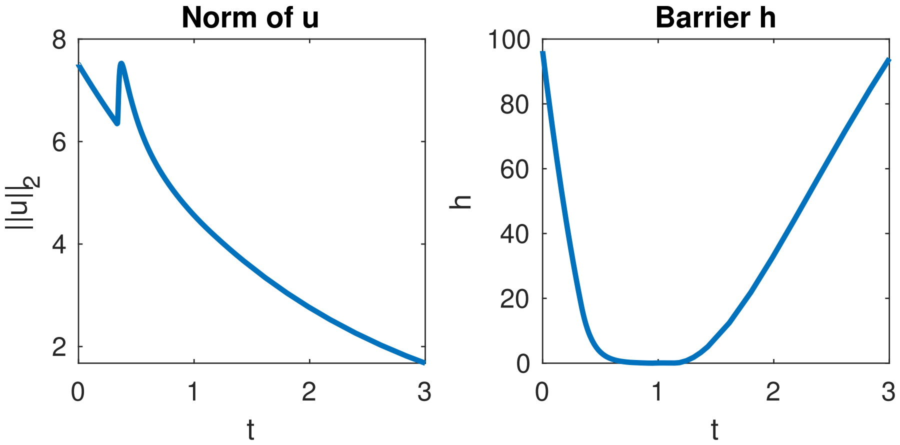


Figure: Trajectory with single obstacle

Simulation results



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Perturbations in safety certificate

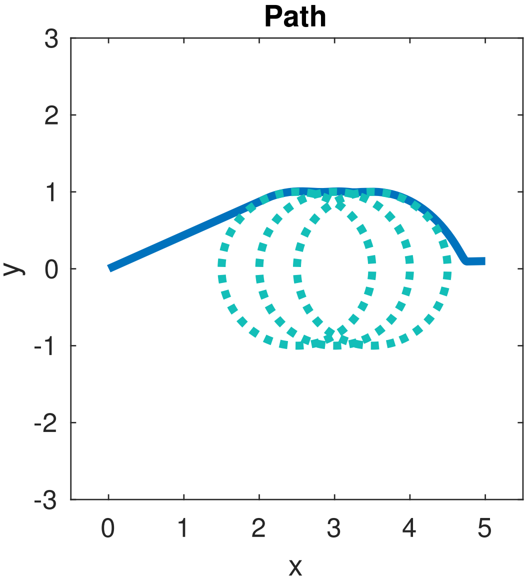
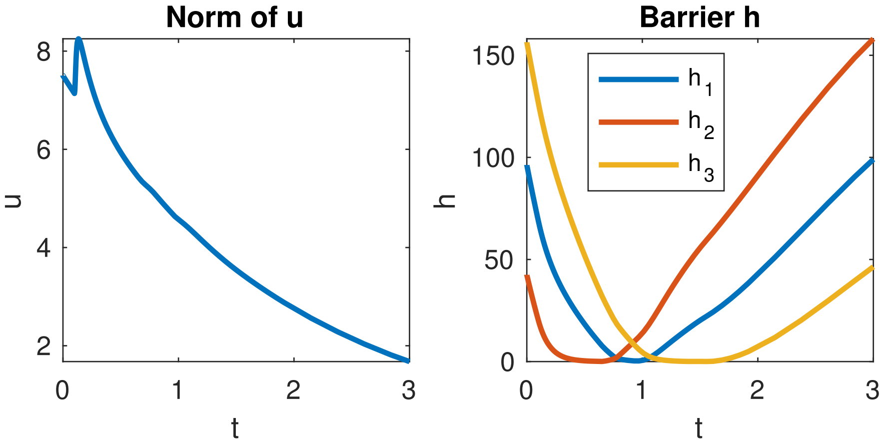


Figure: Trajectory under multiple obstacles

Perturbations in safety certificate



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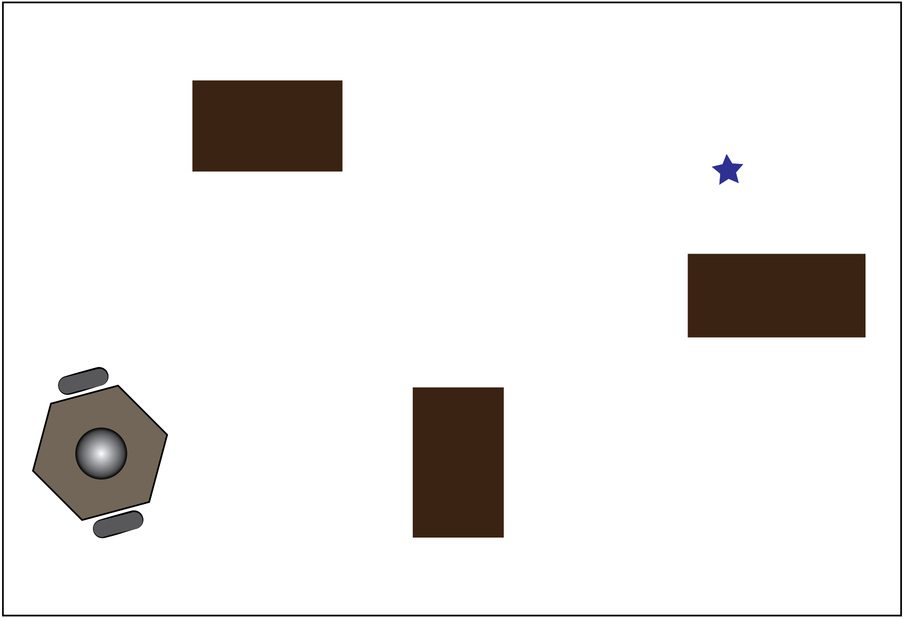
“Almost always” safe against perturbations



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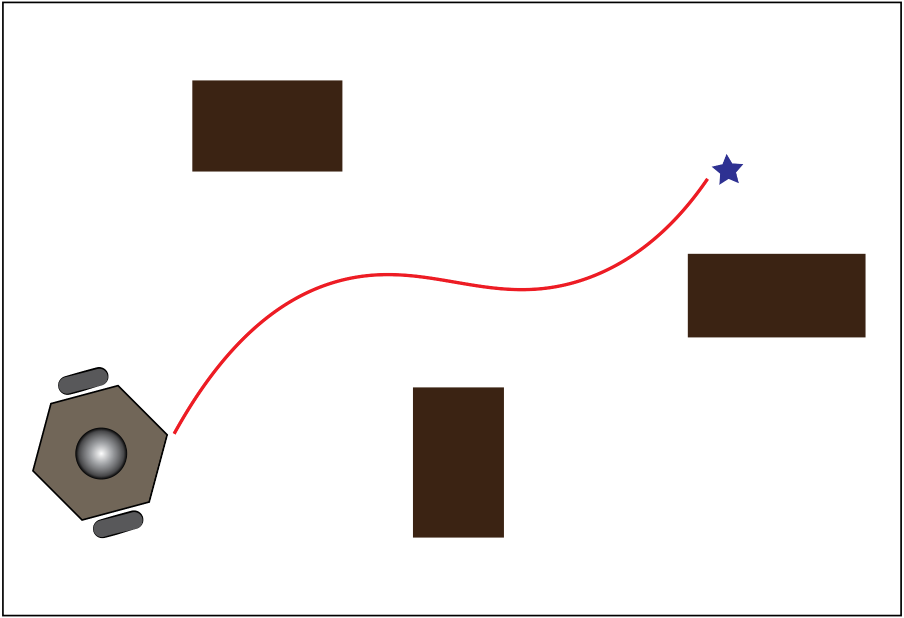

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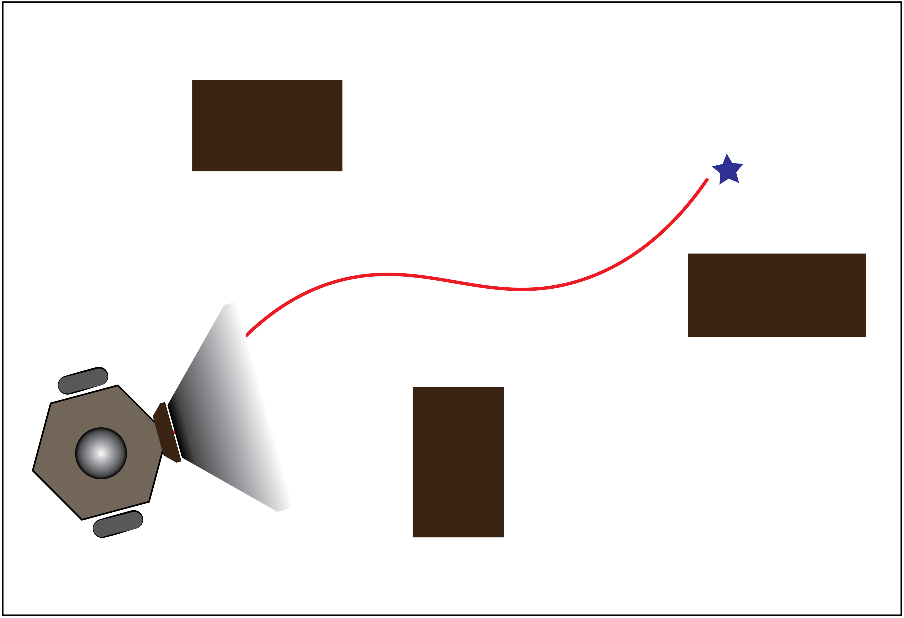

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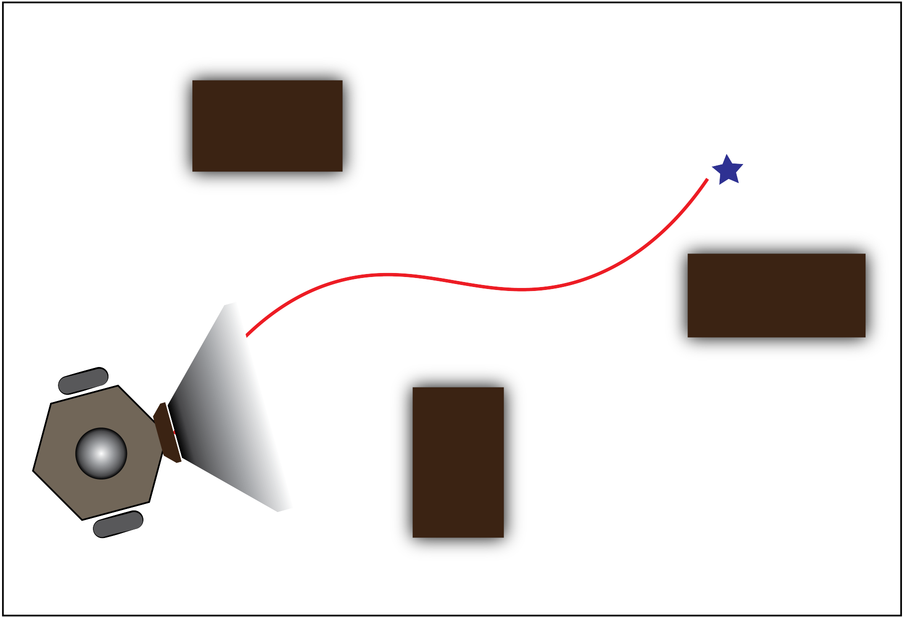

"Almost always" safe against perturbations

$$x(t) \in \mathcal{C} \ \forall t \geq 0$$

and safe-set $\mathcal{C}$ is defined as a super-level set of $h(x) : \mathbb{R}^n \rightarrow \mathbb{R}$, *i.e.*,
$\mathcal{C} = \{x \in \mathbb{R}^n \mid h(x) \geq 0\}$

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- h is function of
  - system states x
  - un-safe regions, *for instance*, $h \equiv dist(x, x_{obstacles}) - \epsilon \geq 0$

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Goal

$$\textbf{Prob}(x \in \mathcal{C}) \geq \eta$$

Stochasticity in Barrier Functions

Control barrier function gives us, $x(t) \in \mathcal{C}$, if

$$\textit{if } \exists u \in \mathcal{U}, \text{ s.t. } \dot{h} + \lambda h \geq 0$$

Probabilistic barrier function formulation

$$\begin{aligned} \min_{u,\delta} \quad & u^\top u + \delta^2 \\ \text{subject to} \quad & L_f V + L_g V u + \gamma V \leq \delta \\ & \textbf{Prob}(L_f h + L_g h u + \lambda h \geq 0) \geq \eta \qquad \text{PrBF} \end{aligned}$$

PrBF depends on choice of h, nature of the probability distribution.

Linear Barrier Function with Normal Distribution

Lets consider a linear barrier function,

$$h(x) = a^\top x + b,$$

for the affine system, $\dot{x} = f(x) + g(x)u$, where $a \sim \mathcal{N}(\bar{a}, \Sigma)$.

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PrBF can be computed as,

$$\begin{aligned} L_f h + L_g h u + \lambda h &= a^\top (f(x) + g(x)u) + \lambda(a^\top x + b) \\ &= a^\top \underbrace{(f(x) + \lambda x + g(x)u)}_{=:-y} + \underbrace{\lambda b}_{=:\tilde{b}} \end{aligned}$$

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Linear Barrier Function with Normal Distribution

Linear Inequalities with Normal Distribution

$$a^\top y \leq b, \quad a \sim \mathcal{N}(\bar{a}, \Sigma)$$

$$\implies a^\top y - b \sim \mathcal{N}(\bar{a}^\top y - b, y^\top \Sigma y)$$

$$\implies \mathbf{Prob}(a^\top y \leq b) = \Phi\left(\frac{b - \bar{a}^\top y}{\sqrt{y^\top \Sigma y}}\right)$$

$$\mathbf{Prob}(a^\top y \leq b) \geq \eta \iff b - \bar{a}^\top y \geq \Phi^{-1}(\eta) \|\Sigma^{1/2} y\|_2$$

Linear Barrier Function with Normal Distribution

Linear Inequalities with Normal Distribution

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$$\implies \textbf{Prob}(a^\top y \leq b) = \Phi\Big(\frac{b - \bar{a}^\top y}{\sqrt{y^\top \Sigma y}}\Big)$$
$$\textbf{Prob}(a^\top y \leq b) \geq \eta \iff b - \bar{a}^\top y \geq \Phi^{-1}(\eta) ||\Sigma^{1/2} y||_2$$

PrBF reformulation into SOCP

$$\left.\begin{array}{rl} \min_{u,\delta} & u^\top u + \delta^2 \\ \text{s.t} & L_f V + L_g V u + \gamma V \leq \delta \\ \textbf{Prob} & \big(L_f h + L_g h u + \lambda h \geq 0\big) \geq \eta \end{array}\right\} \implies \left\{\begin{array}{rl} \min_{u,\delta} & u^\top u^\dagger + \delta^2 \\ \text{s.t} & L_f V + L_g V u + \gamma V \leq \delta \\ & \tilde{b} - \bar{a}^\top y \geq \Phi^{-1}(\eta) ||\Sigma^{1/2} y||_2 \end{array}\right.$$

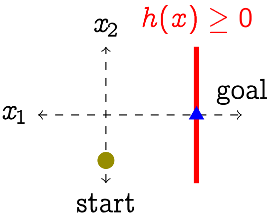
† - Objective function has to be rewritten in terms of y

Example: Linear System with Chance Constraints

- Dynamics: $\dot{x} = Ax + Bu$, with $A = zeros(2,2)$, $B = eye(2)$.
- Barrier function: $h(x) = a^\top x + b$, with $a \sim \mathcal{N}\left(\begin{bmatrix} -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 0.1 \end{bmatrix}\right)$, $b = 4$
- $x_{goal} = \begin{bmatrix} 5 & 0 \end{bmatrix}^\top$

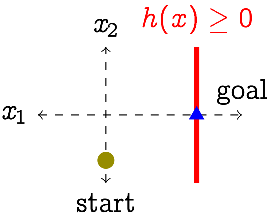
Example: Linear System with Chance Constraints

- Dynamics: $\dot{x} = Ax + Bu$, with $A = zeros(2,2)$, $B = eye(2)$.
- Barrier function: $h(x) = a^\top x + b$, with $a \sim \mathcal{N}\left(\begin{bmatrix} -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 0.1 \end{bmatrix}\right)$, $b = 4$
- $x_{goal} = \begin{bmatrix} 5 & 0 \end{bmatrix}^\top$



Example: Linear System with Chance Constraints

- Dynamics: $\dot{x} = Ax + Bu$, with $A = zeros(2,2)$, $B = eye(2)$.
- Barrier function: $h(x) = a^\top x + b$, with $a \sim \mathcal{N}\left(\begin{bmatrix}-1\\0\end{bmatrix}, \begin{bmatrix}0 & 0\\0 & 0.1\end{bmatrix}\right)$, $b = 4$
- $x_{goal} = \begin{bmatrix}5 & 0\end{bmatrix}^\top$



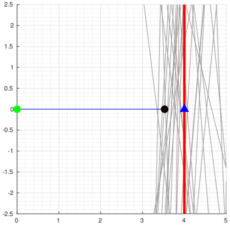
$$L_f h + L_g h u + \lambda h \geq 0 \iff a^\top(Ax + Bu) + \lambda(a^\top x + b) \geq 0$$

$$\textbf{Prob}(L_f h + L_g h u + \lambda h \geq 0) \geq \eta \iff \textbf{Prob}(a^\top y \leq \tilde{b}) \geq \eta,$$

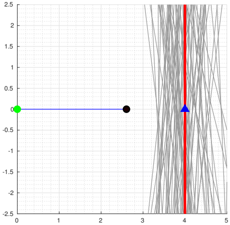
where $y = \underbrace{-((A + \lambda I)x}_{y_0} - Bu$, $\tilde{b} = \lambda b$

$$\begin{array}{ll} \min_{u,\delta} & (y_0 - y)^\top(y_0 - y) + \delta^2 \\ \text{s.t} & -2x^\top P(y) - \delta \leq -(2x^\top P y_0 + 2x^\top PAx + \gamma x^\top Px) \\ & \tilde{b} - \bar{a}^\top y \geq \Phi^{-1}(\eta)\|\Sigma^{1/2} y\|_2 \end{array}$$

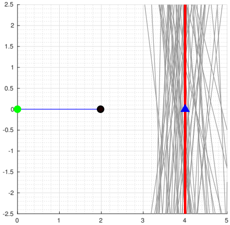
Linear System with Chance Constraints: Result 1



(a) η=0.5, σ=0.1, Failed!

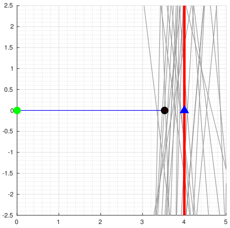


(b) η=0.75, σ=0.1

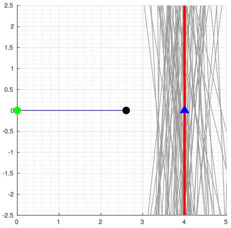


(c) η=0.9, σ=0.1

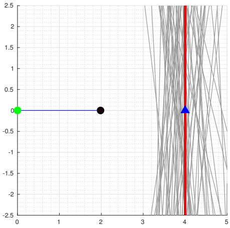
Linear System with Chance Constraints: Result 1



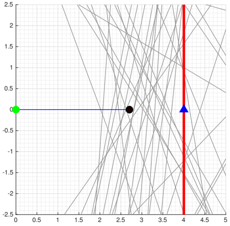
(a) η=0.5, σ=0.1, Failed!



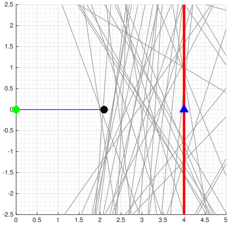
(b) η=0.75, σ=0.1



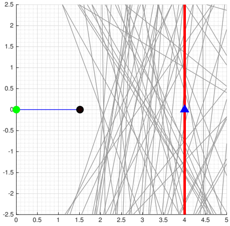
(c) η=0.9, σ=0.1



(d) η=0.75, σ=0.5, Failed!



(e) η=0.9, σ=0.5Failed!



(f) η=0.99, σ=0.5

Figure: Probabilistic Barrier Functions, with $\Sigma = diag([\sigma, \sigma])$

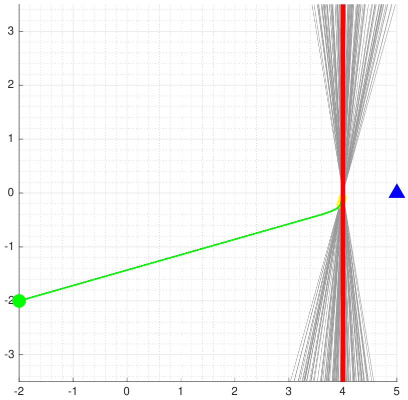
Linear System with Chance Constraints: Result 2

Monte-Carlo simulations:

- η=0.9, $\Sigma = diag([0,\ 0.1])$
- $N = 100$ different simulations (with different random seeds)
- Each iteration simulated for t=0.5s.
- In each iteration, $n = 200$ random values of a are generated (and used during the simulation)

Results:

- # of failures = 13
- Empirical $\mathbf{Prob}(a^\top y \leq \tilde{b}) = 0.87 \approx \eta$



Conclusion and Future Work

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2. Uncertainty in the system dynamics can also be captured via this analysis

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Future work

1. Do away with the linearity assumption in the definition of safe set
2. Analyse higher relative degree systems that is the ones where $L_g h = 0$
3. Accounting for constraints in control inputs